Microeconomic Theory in a Nut Shell

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CONSUMER THEORY: $u(x, y)$ some function such that

$$\frac{\partial u}{\partial x} = MU_x \geq 0, \qquad \underbrace{\frac{\partial^2 u}{\partial x^2} \leq 0}_{\text{law of diminishing marginal utility}}, \qquad \frac{\partial u}{\partial y} = MU_y \geq 0, \qquad \underbrace{\frac{\partial^2 u}{\partial y^2} \leq 0}_{\text{law of diminishing marginal utility}}, \qquad \frac{\partial^2 u}{\partial x \partial y} \gtreqless 0$$

$$u(2 \cdot x, 2 \cdot y) = 2^{\gamma} \cdot u(x, y) \quad u \text{ is homogeneous of degree } \gamma$$

solving u for y yields the indifference curve once a number for u is chosen, say $u = u_0$. $y = f(x \mid u_0)$. The absolute value of the slope of the indifference curve is the Marginal Rate of Substitution (MRS):

$$MRS = \left| \frac{dy}{dx} \right|$$

which is equivalent to

$$0 = du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$MU_x dx + MU_y dy = 0$$

$$\frac{dy}{dx} = -\frac{MU_x}{MU_y}$$

$$MRS = \frac{MU_x}{MU_y}$$

Consumer utility is constrained by the budget:

$$M_0 = P_x x + P_y y \qquad \text{or} \qquad y = \tfrac{M_0}{P_y} - \tfrac{P_x}{P_y} x$$

The absolute value of the budget line (with y on the vertical axis) is sometimes referred to as the Economic Rate of Substitution (ERS):

$$ERS = \frac{P_x}{P_y}$$

The y-intercept is equal to M_0/P_y while the x-intercept is equal to M_0/P_x.

EXAMPLE 1: Suppose consumer utility is given by $u(x,y) = 2x^{0.5} + y^{0.5}$.

$$\begin{aligned} u(4\cdot x, 4\cdot y) &= 2(4\cdot x)^{0.5} + (4\cdot y)^{0.5} \\ &= 2\cdot 2(x)^{0.5} + 2\cdot (y)^{0.5} \\ &= 2\cdot\left(2(x)^{0.5} + (y)^{0.5}\right) \\ &= 4^{0.5}\cdot u(x,y) \end{aligned}$$

Thus the utility function is homogenous of degree 0.5. This means that if the consumer quadruples her consumption of goods x and y her utility doubles. However, if she increases her consumption of goods x and y by a factor of 9, her utility increases by only a factor of 3. The law of diminishing marginal utility holds because

$$\partial u/\partial x = x^{-0.5} > 0 \qquad (\forall x > 0)$$
$$\partial^2 u/\partial x^2 = -0.5x^{-1.5} < 0 \qquad (\forall x > 0)$$

The slope of the indifference curve is

$$MRS = \frac{MU_x}{MU_y} = \frac{x^{-0.5}}{0.5y^{-0.5}} = \frac{2y^{0.5}}{x^{0.5}}$$

Setting this equal to the absolute value of the slope of the budget line (P_x/P_y) yields the optimal decision rule for this consumer:

$$\frac{2y^{0.5}}{x^{0.5}} = \frac{P_x}{P_y}$$

Solving for y yields:

$$y = \frac{P_x^{\,2}}{4P_y^{\,2}}x \qquad (1)$$

Maximize consumer utility given her income is M_0. The Lagrangian is given by

$$\mathcal{L}(x,y,\lambda) = 2x^{0.5} + y^{0.5} + \lambda(M_0 - P_x x - P_y y)$$

The first order conditions (FOCs) are

$$\partial\mathcal{L}/\partial x = x^{-0.5} - \lambda P_x = 0$$
$$\partial\mathcal{L}/\partial y = 0.5y^{-0.5} - \lambda P_y = 0$$
$$\partial\mathcal{L}/\partial\lambda = M_0 - P_x x - P_y y = 0$$

Solving the first two FOCs of λ and then setting the resulting equations equal to each other yields:

$$\frac{x^{-0.5}}{P_x} = \frac{0.5y^{-0.5}}{P_y}.$$

This is says the consumer chooses *x* and *y* so that $MU_x/P_x = MU_y/P_y$. Solving for *y* yields

$$y = \frac{P_x^{\,2}}{4P_y^{\,2}}x,$$

which is identical to equation (1). Substituting this result into the final FOC yields

$$M_0 = P_x x + P_y \frac{P_x^{\,2}}{4P_y^{\,2}}x$$

Solving for *x* yields good x's **Marshallian demand**:

$$x(P_x, P_y, M_0) = \frac{4M_0P_y}{4P_yP_x + P_x^{\,2}}$$

Substituting this into equation (1) yields good y's **Marshallian demand**:

$$y(P_x, P_y, M_0) = \frac{M_0P_x}{4P_y^{\,2} + P_xP_y}$$

Also recall than λ equals $x^{-0.5}/P_x$ and $0.5y^{-0.5}/P_y$. Substituting the appropriate Marshallian into either of these yields

$$\lambda = 0.5\left(\frac{4P_y + P_x}{P_xP_yM_0}\right)^{0.5}$$

Notice that the law of demand holds for the Marshallian demands for goods x and y (i.e., and increase in the price of good x results in less consumption of good x). Also notice that income and prices of related goods shift the Marshallian demands. Finally, since the "4" is the square of the coefficient "2" in the consumer's utility function and this "2" represents consumer preferences, if the consumer's preferences shift, then so do the Marshallian demands. The exponents of *x* and *y* in the consumer's utility function are a result of consumer preferences as well.

Substituting the Marshallian demands into the Lagrangian yields

$$\underbrace{2\left(\frac{4M_0P_y}{4P_yP_x + P_x^{\,2}}\right)^{0.5} + \left(\frac{M_0P_x}{4P_y^{\,2} + P_xP_y}\right)^{0.5}}_{u^*} + \underbrace{\lambda\left[M_0 - P_x\left(\frac{4M_0P_y}{4P_yP_x + P_x^{\,2}}\right) - P_y\left(\frac{M_0P_x}{4P_y^{\,2} + P_xP_y}\right)\right]}_{0}$$

$$= \underbrace{\frac{4\left(M_0P_y\right)^{0.5}}{\left(4P_y+P_x\right)^{0.5}P_x^{0.5}} + \frac{\left(M_0P_x\right)^{0.5}}{\left(4P_y+P_x\right)^{0.5}\left(P_y\right)^{0.5}}}_{u^*} + \lambda\underbrace{\left[M_0 - \frac{4M_0P_y}{4P_y+P_x} - \frac{M_0P_x}{4P_y+P_x}\right]}_{0}$$

$$= \underbrace{\frac{4M_0^{0.5}P_y}{\left(4P_y+P_x\right)^{0.5}\left(P_xP_y\right)^{0.5}} + \frac{M_0^{0.5}P_x}{\left(4P_y+P_x\right)^{0.5}\left(P_yP_x\right)^{0.5}}}_{u^*} + \lambda\underbrace{\left[M_0 - M_0\frac{4P_y+P_x}{4P_y+P_x}\right]}_{0}$$

Which reduces to

$$u^* = \frac{\left(4P_y+P_x\right)^1 M_0^{0.5}}{\left(4P_y+P_x\right)^{0.5}\left(P_xP_y\right)^{0.5}}$$

Further simplification and the replacement of u^* with $v(P_x, P_y, M_0)$ yields the consumer's **indirect utility function**:

$$v(P_x,P_y,M_0) = \left(\frac{\left(4P_y+P_x\right)M_0}{P_xP_y}\right)^{0.5}$$

EXAMPLE 2: Minimize consumer expenditures given utility equal to u_0. The Lagrangian is given by

$$\mathcal{L}(x,y,\lambda) = P_x x + P_y y + \lambda^u\left[u_0 - (2x^{0.5}+y^{0.5})\right]$$

The first order conditions (FOCs) are

$$\partial\mathcal{L}/\partial x = P_x - \lambda^u x^{-0.5} = 0$$
$$\partial\mathcal{L}/\partial y = P_y - 0.5\lambda^u y^{-0.5} = 0$$
$$\partial\mathcal{L}/\partial\lambda^u = u_0 - (2x^{0.5}+y^{0.5}) = 0$$

Notice that solving the first two FOCs for y yields exactly what we found previously:

$$y = \frac{P_x^{\,2}}{4P_y^{\,2}}x\,,$$

only this time we substitute this result into a different final FOC:

$$u_0 = 2x^{0.5} + \left(\frac{P_x^{\,2}}{4P_y^{\,2}}x\right)^{0.5}$$

Solving for *x* yields good x's **Hicksian (or compensated) demand**:

$$x^c(P_x, P_y, u_0) = \frac{4P_y^{\,2} u_0^{\,2}}{\left(4P_y + P_x\right)^2}$$

Substituting this into equation (1) yields good y's **Hicksian (or compensated) demand**:

$$y^c(P_x, P_y, u_0) = \frac{P_x^{\,2} u_0^{\,2}}{\left(4P_y + P_x\right)^2}\,.$$

Notice that the law of demand holds for the Hicksian demands for goods x and y. Also notice that utility, prices of related goods, and the relative preference of good x to y shift these demands (the coefficient of 2 in the utility function and the exponents of *x* and *y*).

Also recall than λ^u equals $P_x x^{0.5}$ and $0.5P_y y^{0.5}$. Substituting the appropriate Hicksian into either of these yields

$$\lambda^u = \frac{2P_x P_y}{4P_y + P_x} u_0$$

Notice that if we substitute $v(P_x, P_y, M_0)$ in for u_0 into λ^u the inverse of λ:

$$\lambda^u = \frac{2P_x P_y}{4P_y + P_x}\left(\frac{\left(4P_y + P_x\right)M_0}{P_x P_y}\right)^{0.5}$$

$$\lambda^u = \frac{2\left(P_x P_y M_0\right)^{0.5}}{\left(4P_y + P_x\right)^{0.5}} = \lambda^{-1}\,.$$

Substituting the Hicksian demands into the Lagrangian yields

$$\underbrace{P_x \frac{4P_y^{\,2} u_0^{\,2}}{\left(4P_y + P_x\right)^2} + P_y \frac{P_x^{\,2} u_0^{\,2}}{\left(4P_y + P_x\right)^2}}_{Expenditures} + \underbrace{\lambda^u \left\{ u_0 - \left[2\left(\frac{4P_y^{\,2} u_0^{\,2}}{\left(4P_y + P_x\right)^2}\right)^{0.5} + \left(\frac{P_x^{\,2} u_0^{\,2}}{\left(4P_y + P_x\right)^2}\right)^{0.5} \right] \right\}}_{0}$$

$$= \underbrace{\left(\frac{4P_y}{\left(4P_y + P_x\right)^2} + \frac{P_x}{\left(4P_y + P_x\right)^2}\right) P_x P_y u_0^{\,2}}_{Expenditures} + \underbrace{\lambda^u \left(u_0 - u_0 \frac{4P_y + P_x}{4P_y + P_x} \right)}_{0}$$

Further simplification yields the optimized consumer **expenditure function**:

$$E(P_x, P_y, u_0) = \frac{P_x P_y u_0^{\,2}}{4P_y + P_x}$$

The power of the expenditure function allows an economist to compute the income necessary to make the consumer indifferent (*u* does not change) after a change in say the price of good x (the price of y doesn't change but x's does from P_x to P'_x):

$$\Delta M = E(P'_x, P_y, u_0) - E(P_x, P_y, u_0) = \frac{P'_x P_y u_0^{\,2}}{4P_y + P'_x} - \frac{P_x P_y u_0^{\,2}}{4P_y + P_x}$$

Also, differentiating the expenditure function with respect to the price of good x yields the Hicksian demand equation (**Shephard's Lemma**):

$$\frac{\partial E(P_x, P_y, u_0)}{\partial P_x} = \frac{4P_y P_y u_0^{\,2}}{\left(4P_y + P_x\right)^2} = x^c(P_x, P_y, u_0)$$

An example of a compensating income differential was the proposal by the Senate Republicans to compensate commuters with a one-time $100 payment after the spike in gas prices in the spring of 2006.

Duality between the two problems (utility max and cost min) is demonstrated by solving $E(P_x, P_y, u_0) = M_0$ for u_0 and then solve $v(P_x, P_y, M_0) = u_0$ for M_0:

$$M_0 = E(P_x, P_y, u_0)$$

$$M_0 = \frac{P_x P_y u_0^{\,2}}{4P_y + P_x}$$

$$\frac{(4P_y + P_x)M_0}{P_x P_y} = u_0^{\,2}$$

$$v(P_x, P_y, M_0) = \left(\frac{\left(4P_y + P_x\right)M_0}{P_x P_y}\right)^{0.5}$$

Properties of consumer theory equations:

(1) $\frac{\partial E(P_x, P_y, u_0)}{\partial P_x} \geq 0$ which means $P_x \uparrow \Rightarrow E(P_x, P_y, u_0) \uparrow$

(2) $E(P_x, P_y, u_0)$ is concave in P_x.

(3) Property (2) is equivalent to "the Law of Demand" holding for good *x*:

$$\frac{\partial E(P_x, P_y, u_0)}{\partial P_x} = x^c(P_x, P_y, u_0) \qquad \text{and} \qquad \frac{\partial^2 E(P_x, P_y, u_0)}{\partial P_x^{\ 2}} = \frac{\partial x^c(P_x, P_y, u_0)}{\partial P_x} \le 0$$

(6) $E(2 \cdot P_x, 2 \cdot P_y, u_0) = 2 \cdot E(P_x, P_y, u_0)$

(7) $x(2 \cdot P_x, 2 \cdot P_y, 2 \cdot M_0) = x(P_x, P_y, M_0)$

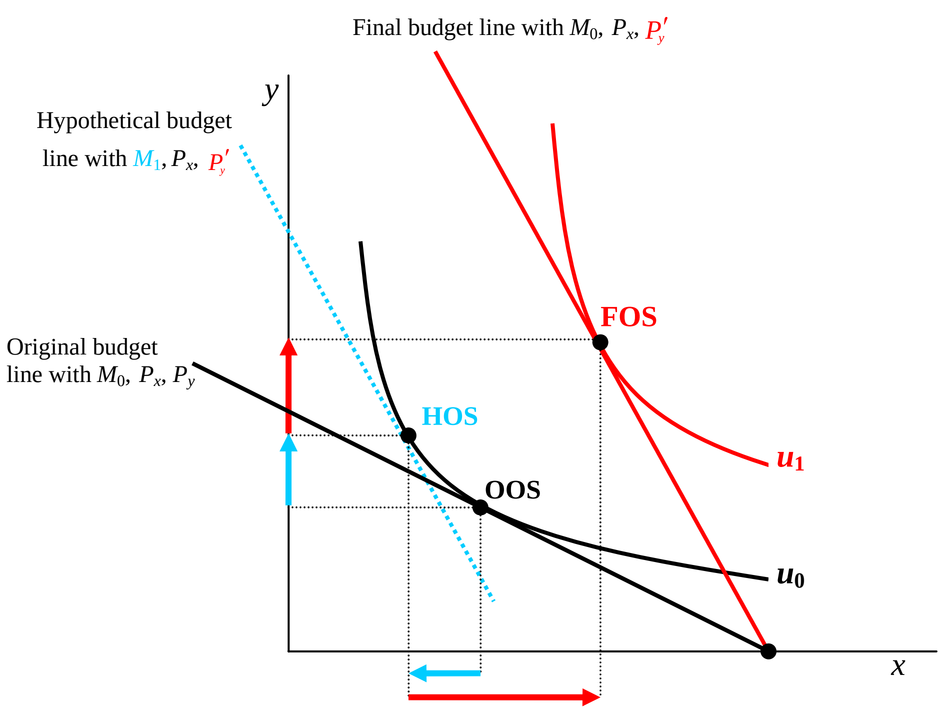
(8) $x^c\left(P_x, P_y, v(P_x, P_y, M_0)\right) = x\left(P_x, P_y, M_0\right)$

(9) $x\left(P_x, P_y, E(P_x, P_y, u_0)\right) = x^c\left(P_x, P_y, u_0\right)$

(10) $x^c(2 \cdot P_x, 2 \cdot P_y, u_0) = x^c(P_x, P_y, u_0)$

Using the Marshallian and Hicksian demands and the indirect utility and expenditure functions, verify all of the properties above.

Suppose the price of good *y* **decreases** from P_y to P_y':



$$M_0 > M_1 = M_0 - \Delta M \quad \text{and} \quad P_y > P_y'$$

OOS:	$u_0 = v(P_x, P_y, M_0)$	$M_0 = E(P_x, P_y, u_0)$
HOS:	$u_0 = v(P_x, P_y', M_1)$	$M_1 = E(P_x, P_y', u_0)$
FOS:	$u_1 = v(P_x, P_y', M_0)$	$M_0 = E(P_x, P_y', u_1)$

OOS:	$x(P_x, P_y, M_0) = x^c(P_x, P_y, u_0)$
HOS:	$x(P_x, P_y', M_1) = x^c(P_x, P_y', u_0)$
FOS:	$x(P_x, P_y', M_0) = x^c(P_x, P_y', u_1)$

Derive the **Slutsky equation** by differentiating x^c with respect to P_x using the chain rule:

$$x^c\left(P_x, P_y, u_0\right) = x\left(P_x, P_y, \underbrace{E(P_x, P_y, u_0)}\right)$$

Argument M of the Marshallian Demand for good x

$$\frac{\partial x^c\left(P_x, P_y, u_0\right)}{\partial P_x} = \frac{\partial x\left(P_x, P_y, E(P_x, P_y, u_0)\right)}{\partial P_x} + \frac{\partial x\left(P_x, P_y, E(P_x, P_y, u_0)\right)}{\partial M} \cdot \frac{\partial E(P_x, P_y, u_0)}{\partial P_x}$$

$$\frac{\partial x^c\left(P_x, P_y, u_0\right)}{\partial P_x} = \frac{\partial x\left(P_x, P_y, M_0\right)}{\partial P_x} + \frac{\partial x\left(P_x, P_y, M_0\right)}{\partial M} \cdot \frac{\partial E(P_x, P_y, u_0)}{\partial P_x}$$

$$\underbrace{\frac{\partial x}{\partial P_x}}_{TE_x} = \underbrace{\frac{\partial x^c}{\partial P_x}}_{SE_x} - \underbrace{\frac{\partial x}{\partial M}\frac{\partial E}{\partial P_x}}_{IE_x}$$

$$SE_x = x(P_x, P_y', M_1) - x(P_x, P_y, M_0)$$
$$SE_x = x^c(P_x, P_y', u_0) - x^c(P_x, P_y, u_0)$$

$$TE_x = x(P_x, P_y', M_0) - x(P_x, P_y', M_1)$$
$$TE_x = x^c(P_x, P_y, u_1) - x^c(P_x, P_y', u_0)$$

$$TE_x = x(P_x, P_y', M_0) - x(P_x, P_y, M_0)$$
$$TE_x = x^c(P_x, P_y', u_1) - x^c(P_x, P_y', u_0)$$

LONG RUN FIRM THEORY: $q(L, K)$ some function such that

$$\frac{\partial q}{\partial L} = MP_L \geq 0, \quad \underbrace{\frac{\partial^2 q}{\partial L^2} \leq 0}_{\text{law of diminishing marginal productivity of Labor}}, \quad \frac{\partial q}{\partial K} = MP_K \geq 0, \quad \underbrace{\frac{\partial^2 q}{\partial K^2} \leq 0}_{\text{law of diminishing marginal productivity of capital}}, \quad \frac{\partial^2 q}{\partial L \partial K} \geq 0$$

$$q(2 \cdot L, 2 \cdot K) = 2^{\gamma} \cdot q(L, K) \qquad \begin{cases} \text{DRS if} & \gamma < 1 \\ \text{CRS if} & \gamma = 1 \\ \text{IRS if} & \gamma > 1 \end{cases}$$

solving q for K yields the isoquant curve once a number for q is chosen, say $q = q_0$. $K = f(L \mid q_0)$. The absolute value of the slope of the isoquant is the Marginal Rate of Technical Substitution (*MTRS*):

$$MRTS = \left| \frac{dK}{dL} \right|$$

which is equivalent to

$$0 = dq = \frac{\partial q}{\partial K} dK + \frac{\partial q}{\partial L} dL$$

$$MU_K dK + MP_L dL = 0$$

$$\frac{dK}{dL} = -\frac{MP_L}{MP_K}$$

$$MRTS = \frac{MP_L}{MP_K}$$

If the firm produces q units of output it requires K and L units of capital and labor which costs

$$C_0 = wL + rK \qquad \text{or} \qquad K = \tfrac{C_0}{r} - \tfrac{w}{r} L$$

The absolute value of the isocost is sometimes referred to as the Economic Rate of Technical Substitution (*ERTS*):

$$ERTS = \frac{w}{r}$$

The K-intercept is equal to C_0/r while the L-intercept is equal to C_0/w.

EXAMPLE 3: Suppose the firm's production function is given by

$$q(L, K) = Ae^{0.1t}\left(2L^{0.5} + K^{0.5}\right)$$

At time $t = 0$, this simplifies to

$$q(L, K) = 2L^{0.5} + K^{0.5}$$

if A is equal to 1. Thus the production function is homogenous of degree 0.5, which means the production function exhibits decreasing returns to scale:

$$q(4 \cdot x, 4 \cdot y) = 2(4 \cdot L)^{0.5} + (4 \cdot K)^{0.5}$$

$$= 2 \cdot 2(L)^{0.5} + 2 \cdot (K)^{0.5}$$
$$= 2 \cdot \left(2(L)^{0.5} + (K)^{0.5}\right)$$
$$= 4^{0.5} \cdot q(L,K)$$

The law of diminishing marginal productivity of labor holds because

$$\partial q / \partial L = L^{-0.5} > 0 \qquad (\forall L > 0)$$
$$\partial^2 q / \partial L^2 = -0.5L^{-1.5} < 0 \qquad (\forall L > 0)$$

The slope of the isoquant curve is

$$MRTS = \frac{MP_L}{MP_K} = \frac{L^{-0.5}}{0.5K^{-0.5}} = \frac{2K^{0.5}}{L^{0.5}}$$

Setting this equal to the absolute value of the slope of the isocost line (w/r) yields the optimal decision rule for this firm:

$$\frac{2K^{0.5}}{L^{0.5}} = \frac{w}{r}$$

Solving for K yields:

$$K = \frac{w^2}{4r^2} L . \qquad (2)$$

Notice that equation (2) is analogous to equation (1). Over time (t increases) the capital and labor that is necessary to produce a given level of output (q_0) reduce because an increase in t shifts the isoquant corresponding to q_0 in toward the origin.

Over the long run firms choose capital and labor to minimize the cost of producing given output equal to q_0. The Lagrangian is given by

$$\mathcal{L}(L,K,\lambda) = wL + rK + \lambda^u \left[q_0 - (2L^{0.5} + K^{0.5}) \right]$$

The above problem assumes the firm competes in perfectly competitive factor markets (K and L) because w and r are given to the firm. That is, w is not a function of L and r is not a function of K. The first order conditions (FOCs) are

$$\partial \mathcal{L} / \partial L = w - \lambda^u L^{-0.5} = 0$$
$$\partial \mathcal{L} / \partial K = r - 0.5\lambda^u K^{-0.5} = 0$$
$$\partial \mathcal{L} / \partial \lambda^u = q_0 - (2L^{0.5} + K^{0.5}) = 0$$

Notice that solving the first two FOCs for y yields exactly what we found previously in equation (2):

$$K = \frac{w^2}{4r^2} L \,,$$

only this time we substitute this result into a different final FOC:

$$q_0 = 2L^{0.5} + \left(\frac{w^2}{4r^2} L \right)^{0.5}$$

Solving for *L* yields labor's **contingent factor demand**:

$$L^c(w, r, q_0) = \frac{4r^2 q_0^{\,2}}{(4r + w)^2}$$

Substituting this into equation (2) yields capital's **contingent factor demand**:

$$K^c(w, r, q_0) = \frac{w^2 q_0^{\,2}}{(4r + w)^2} \,.$$

Notice that the law of demand holds for the contingent factor demands for capital and labor (e.g., higher *w* results in less *L*). Also notice that quantity, prices of other inputs, and the relative productivity of labor to capital (coefficient 2 in the output function and the exponents of *L* and *K*) shift these demands. Finally, it can be shown that over time the contingent factor demand for capital decreases.

Also recall than λ^u equals $wL^{0.5}$ and $2rK^{0.5}$. Substituting the appropriate contingent factor demand into both of these yields

$$\lambda^u = \frac{2rwq_0}{4r + w}$$

Substituting the contingent factor demands into the Lagrangian yields

$$\underbrace{w\frac{4r^2 q_0^{\,2}}{(4r+w)^2} + r\frac{w^2 q_0^{\,2}}{(4r+w)^2}}_{cost} + \underbrace{\lambda^u \left\{ q_0 - \left[2\left(\frac{4r^2 q_0^{\,2}}{(4r+w)^2} \right)^{0.5} + \left(\frac{w^2 q_0^{\,2}}{(4r+w)^2} \right)^{0.5} \right] \right\}}_{0}$$

$$= \underbrace{\left(\frac{4r}{(4r+w)^2} + \frac{w}{(4r+w)^2} \right) wrq_0^{\,2}}_{cost} + \underbrace{\lambda^u \left(q_0 - q_0 \frac{4r+w}{4r+w} \right)}_{0}$$

Further simplification yields the optimized firm **cost function**:

$$C(w,r,q)=\frac{wrq^2}{4r+w}.$$

Notice that I replaced q_0 with simply q. Differentiating this with respect to q yields the firm's long run **marginal cost function**:

$$MC(w,r,q)=\frac{2wrq}{4r+w}$$

Dividing the cost equation by q yields the firm's long run **average cost function**:

$$AC(w,r,q)=\frac{wrq}{4r+w}$$

Differentiating the cost function with respect to the factor prices yields the contingent factor demand equations (**Shephard's Lemma**):

$$\frac{\partial C(w,r,q)}{\partial w}=\frac{4r^2q^2}{\left(4r+w\right)^2}=L^c(w,r,q)$$

$$\frac{\partial C(w,r,q)}{\partial r}=\frac{w^2q^2}{\left(4r+w\right)^2}=K^c(w,r,q)$$

PROBLEM 4: The following profit maximizing problem assumes the firm competes in perfectly competitive output (q) and factor markets (K and L) because w, r and p are given to the firm. That is, w is not a function of L, r is not a function of K and p is not a function of q.

$$\max \pi(L,K)=\underbrace{p\cdot\left(2L^{0.5}+K^{0.5}\right)}_{revenue}-\underbrace{\left(w\cdot L+r\cdot K\right)}_{cost}$$

The FOC are

$$\pi_L(L,K)=\underbrace{p\cdot L^{-0.5}}_{MRP_L}-w=0$$

$$\pi_K(L,K)=\underbrace{0.5\cdot p\cdot K^{-0.5}}_{MRP_K}-r=0$$

Notice that solving the first two FOCs for y yields exactly what we found previously:

$$K=\frac{w^2}{4r^2}L,$$

only this time we substitute this result into the second profit max FOC, and then solve this for L. Thus the **factor demand** for labor is given by

$$L(w,r,p) = \frac{p^2}{w^2}.$$

Substituting this into equation (2) yields good capital's **factor demand**:

$$K(w,r,p) = \frac{p^2}{4r^2}.$$

Notice that the law of demand holds for the factor demands for capital and labor (e.g., higher *w* results in less *L*). Also notice that the relative productivity of labor to capital (coefficient 2 in the output function and the exponents of *L* and *K*) shifts these demands. Finally, it can be shown that over time the contingent factor demand for capital decreases due to increases in technology.

Substituting the factor demands results into the firms profit equation yields

$$p \cdot \left(2\left(\frac{p^2}{w^2}\right)^{0.5} + \left(\frac{p^2}{4r^2}\right)^{0.5} \right) - w\left(\frac{p^2}{w^2}\right) - r\left(\frac{p^2}{4r^2}\right)$$
$$= p \cdot \left(2\frac{p}{w} + \frac{p}{2r} \right) - \left(\frac{p^2}{w} + \frac{p^2}{4r} \right)$$
$$= \frac{p^2}{w}\frac{4r}{4r} + \frac{p^2}{4r}\frac{w}{w}$$

Further simplification yields the optimized firm **profit function**:

$$\Pi(r,w,p) = \frac{4r+w}{4rw}p^2.$$

The profit function is very powerful. Differentiating the profit function with respect to the *p*, *r*, and *w* yield the **supply function** and the negative of the factor demands for capital and labor, respectively, (**Envelope Theorem**):

$$\frac{\partial \Pi(r,w,p)}{\partial p} = \frac{(4r+w)p}{4rw} = q(r,w,p)$$

$$\frac{\partial \Pi(r,w,p)}{\partial r} = \frac{-p^2}{4r^2} = -K(r,w,p)$$

$$\frac{\partial \Pi(r,w,p)}{\partial w} = \frac{-p^2}{w^2} = -L(r,w,p)$$

In a perfectly competitive market the firm cannot influence the price of its output. If it tries to sell above the going price, it sells nothing. But, it can sell all it wants at the going price, but it maximizes profit when it chooses an output level such that MR equals MC:

$$p = MC(w,r,q)$$

Duality between the two problems (profit max and cost min) is demonstrated by solving $q(w,r,p) = q_0$ for q_0 and then solving $MC(w,r,q_0) = p$ for p: Solving $p = MC(w,r,q)$ for q yields the supply equation:

$$p = MC(w,r,q)$$

$$p = \frac{2wrq}{4r + w}$$

$$q(w,r,p) = \frac{2wrp}{4r + w}$$

Thus the MC equation is the firm's inverse supply equation. An increase in MC is a decrease in supply, a decrease in MC is an increase in supply.

Properties of firm theory equations:

(1) $\frac{\partial C(w,r,q_0)}{\partial w} \geq 0$, $\frac{\partial C(w,r,q_0)}{\partial r} \geq 0$, $\frac{\partial C(w,r,q)}{\partial q} = MC \geq 0$ which mean $w\uparrow, r\uparrow, q\uparrow \Rightarrow C(w,r,q_0)\uparrow$

(2) $C(w,r,q_0)$ is concave in w.

(3) Property (2) is equivalent to "the Law of Demand" holding for L:

$$\frac{\partial C(w,r,q_0)}{\partial w} = L^c(w,r,q_0) \quad \text{and} \quad \frac{\partial C(w,r,q_0)}{\partial w} = \frac{\partial L^c(w,r,q_0)}{\partial w} \leq 0$$

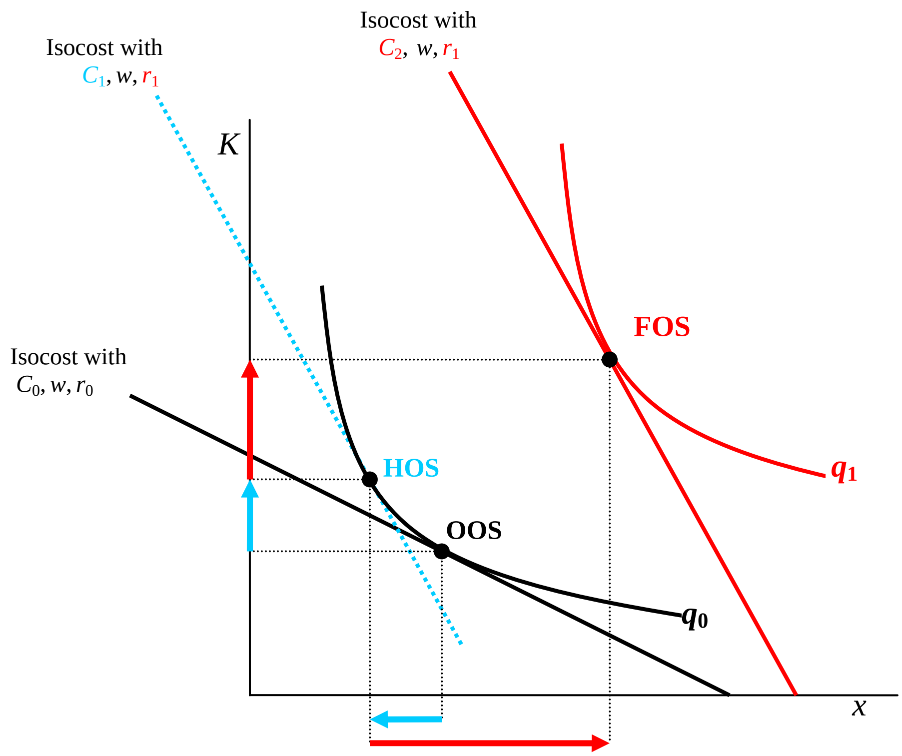
(4) $C(2\cdot w, 2\cdot r, q_0) = 2\cdot C(w,r,q_0)$
$MC(2\cdot w, 2\cdot r, q_0) = 2\cdot MC(w,r,q_0)$
$AC(2\cdot w, 2\cdot r, q_0) = 2\cdot AC(w,r,q_0)$

(5) $L^c\left(w,r,q(w,r,P)\right) = L\left(w,r,P\right)$

(6) $L\left(w,r,MC(w,r,q_0)\right) = L^c\left(w,r,q_0\right)$ in a perfectly competitive market

Suppose rent (r) **decreases** from r_0 to r_1:

$$C_1 < C_0 < C_2 \quad \text{and} \quad r_0 > r_1$$



OOS: $L^c(w, r_0, q_0)$ and $K^c(w, r_0, q_0)$
HOS: $L^c(w, r_1, q_0)$ and $K^c(w, r_1, q_0)$
FOS: $L^c(w, r_1, q_1)$ and $K^c(w, r_1, q_1)$

$$SE_L = L^c(w, r_1, q_0) - L^c(w, r_0, q_0)$$
$$SE_K = K^c(w, r_1, q_0) - K^c(w, r_0, q_0)$$

$$OE_L = L^c(w, r_1, q_1) - L^c(w, r_1, q_0)$$
$$OE_K = K^c(w, r_1, q_1) - K^c(w, r_1, q_0)$$

$$TE_L = L^c(w, r_1, q_1) - L^c(w, r_0, q_0)$$
$$TE_K = K^c(w, r_1, q_1) - K^c(w, r_0, q_0)$$

SHORT RUN FIRM THEORY: K is fixed in the short-run, say at $\bar{K} = 36$. Therefore, at time $t = 0$ with $A = 1$ and $r = 100$ the short-run production function is given by

$$q(L,\bar{K}) = Ae^{0.1t}\left(2L^{0.5} + \bar{K}^{0.5}\right)$$

$$q(L) = 6 + 2L^{0.5}.$$

PROBLEM 5: The price taking, profit maximizing firm chooses L to solve

$$\max \pi(L) = \underbrace{p\cdot\left(6 + 2L^{0.5}\right)}_{Rev} - \underbrace{w\cdot L}_{VC} - \underbrace{100\cdot 36}_{FC}$$

Step 1: Compute the FOC (one equation and one unknown):

$$\pi_L(L) = 0 \qquad \Leftrightarrow \qquad p \cdot MP_L(L) = w$$

Step 2: The equation $w = p \cdot MP_L(L)$ is the short run labor demand equation.

PROBLEM 6: Solve $q(L) = 6 + 2L^{0.5}$ for *L* which yields a function in *q*:

$$L(q) = 0.25q^2 - 3q + 9$$

The profit maximizing problem with choice *L*:

$$\max \pi(q) = \underbrace{p \cdot q}_{Rev} - \underbrace{w \cdot \left(0.25q^2 - 3q + 9\right)}_{VC} - \underbrace{100 \cdot 36}_{FC}$$

Step 1: Compute the FOC (one equation and one unknown):

$$\pi_q(q) = 0 \qquad \Leftrightarrow \qquad MC(q) = p$$

Step 2: The equation $p = MC(q)$ is the short-run supply equation.

AGGREGATION: Let q^D equal the quantity of good *x* demanded, let *p* be the price of good *x*, and let *N* be the number of consumers in this market

$$Q^D = N \cdot x(p, P_y, M_0)$$

$$Q^D(p, P_y, M_0, N) = \frac{4NM_0P_y}{4P_y p + p^2}$$

Notice that we used the Marshallian demand equation for good x. We use this one rather than Hicksian because the Marshallian is observable with firm level date (OOS to FOS). That is the consumer reveals her preferences with the purchases she makes. However, the Hicksian could be derived via a survey. State preferences are captured with surveys. Thus we would use the Hicksian to aggregate from stated preference studies. Suppose there are *n* firms. Thus, the market supply equation is

$$Q^S = n \cdot q(w, r, p)$$

$$Q^S(w, r, p, n) = \frac{2nwrp}{4r + w}$$

Notice that the laws of demand and supply hold for the aggregated demand and supply equations, respectively. An increase in the price of good x (*p*) increase the quantity supplied (Q^S) but reduces the quantity demanded (Q^D). Shifters of aggregate supply include the number of firms (n), and the prices of inputs (w and r). Less obvious shifters of aggregate supply include the relative productivity of labor to capital (coefficient 2 in the output function and the exponents of L and K) increases in technology ($Ae^{\alpha t}$). Shifters of aggregate demand include the population ($\underline{N}$), prices of other goods (P_y), consumer income (M_0). A less obvious shifter of aggregate demand includes consumer preferences (coefficient 2 in the utility function and the exponents of x and y).

EQUILIBRIUM UNDER PERFECT COMPETITION: Given the price of good y (P_y), number of firms (n), number of consumers (N), consumer income (M_0), wage (w), and rent (r), the equilibrium price of good x is the solution of setting Q^D equal to Q^S:

$$Q^S = Q^D$$

$$\frac{2nwrp}{4r+w} = \frac{4NM_0P_y}{4P_y p + p^2}$$

$$p^3 + 4P_y p^2 - \frac{NM_0P_y}{nwr}(4r+w) = 0\,.$$

The solution to the above cubic polynominal (p^*) has three values, two of which are probably either complex or negative roots. We would simply pick the positive valued p^*. For example suppose there are 1,000,000 consumers, 100 firms, P_y = \$2, w = \$16 per hour, r = \$60 per hour, and M = \$2000. Then p^* equals \$274.70 or -141.34 ± 240.16i. Thus p^* = \$274.70. Substituting this into either Q^D or Q^S:

$$Q^* = 206{,}025 \quad \text{(units sold).}$$

MONOPOLY EQUILIBRIUM: There is only one firm (n = 1). Note because there is only one firm in this market, q_0 in the cost equation becomes Q:

$$C = C(w, r, Q)\,.$$

Given values for wages (w) and rents (r), the cost equation simplifies to

$$C = C(Q)\,.$$

Solving $Q^D = N \cdot x(P, P_y, M_0)$ for P yields the inverse market demand equation, say

$$P = P\left(Q, N, P_y, M_0\right).$$

Given values for the price of good y (P_y), number of consumers (N) and consumer income (M_0) the inverse demand equation simplifies to

$$P = P\left(Q\right).$$

The monopolist's problem is

$$\max \pi(Q) = P\left(Q\right)\cdot Q - C(Q)$$

Step 1: Compute the FOC (one equation and one unknown):

$$\pi_Q(Q) = 0 \quad \Leftrightarrow \quad \underbrace{\frac{\partial P\left(Q\right)}{\partial Q}\cdot Q + P\cdot\frac{\partial Q}{\partial Q}}_{MR(Q)} = \underbrace{\frac{\partial C(Q)}{\partial Q}}_{MC(Q)},$$

If inverse market demand curve is for example equal to $P(Q) = 1000 - 2Q$, then

$$\frac{\partial P}{\partial Q} = -2$$

$$MR(Q) = \left(\tfrac{\partial P}{\partial Q}\right)\cdot Q + \left\{P\right\}$$

$$MR(Q) = \left(-2\right)\cdot Q + \left\{1000 - 2Q\right\},$$

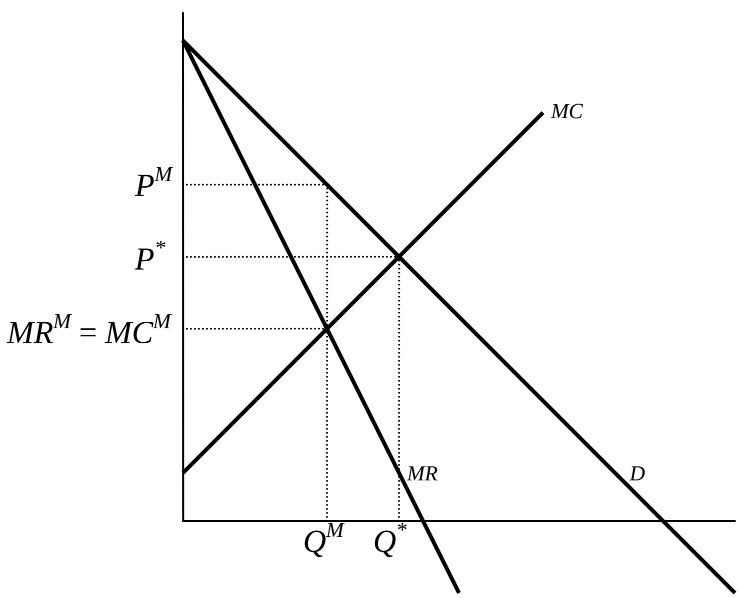
$$MR(Q) = 1000 - 4Q$$

This is identical to differentiating the monopolist revenue equation $R(Q) = 1000Q - 2Q^2$:

$$\tfrac{dR}{dQ} = 1000 - 4Q$$

Step 2: The FOC is equivalent to $MR = MC$. Solving this one equation for the one variable Q yields the monopolist's output Q^M.

Step 3: Substitute Q^M into the inverse market demand curve because the monopolist can charge up to the market demand curve. This yields the monopolist's price P^M.



Monopoly Example: Suppose there is one firm in a market and the **market inverse demand curve** and the monopolist's **cost curve** are given by

$$p(Q) = 250 - 3Q$$

$$C(Q) = 1250 + 50 \cdot Q + 0.5Q^2$$

The monopoly chooses Q to maximize its profit:

$$\max \pi(Q) = p(Q) \cdot Q - C(Q)$$

The general FOC is

$$\pi_Q = \underbrace{\tfrac{dp(Q)}{dQ} \cdot Q + p(Q) \cdot \tfrac{dQ}{dQ}}_{MR} - MC(Q) = 0$$

$$\tfrac{dp}{dQ} \cdot Q \tfrac{p}{p} \cdot + p = MC$$

$$\tfrac{dp}{dQ} \cdot \tfrac{Q}{p} \cdot p = MC - p$$

$$\frac{1}{\varepsilon} \equiv \frac{dp}{dQ} \cdot \frac{Q}{p} = \frac{MC - p}{p}$$

$$-\frac{1}{\varepsilon} = \frac{p - MC}{p} \equiv \text{Lerner Index } (LI)$$

The monopolist's revenue equation is

$$R = \left(p\right) \cdot Q = \left(250 - 3Q\right) \cdot Q = 250Q - 3Q^2$$

The monopoly chooses Q to maximize its profit:

$$\max \pi(Q) = 250Q - 3Q^2 - \left[1250 + 50 \cdot Q + 0.5Q^2\right]$$

The FOC is

$$\pi_Q = \underbrace{250 - 6Q}_{MR} - \underbrace{\left[50 + Q\right]}_{MC \text{ or } mkt\ S} = 0$$

$$250 - 6Q = 50 + Q$$

$$250 - 50 = 7Q$$

$$Q^M = \tfrac{200}{7} = 28.57$$

$$MC^M = 50 + 28.57 = \$78.57$$

$$MR^M = 250 - 6 \cdot 28.57 = \$78.57$$

$$p^M = 250 - 3 \cdot 28.57 = 164.29$$

The Lerner Index for the monopoly is

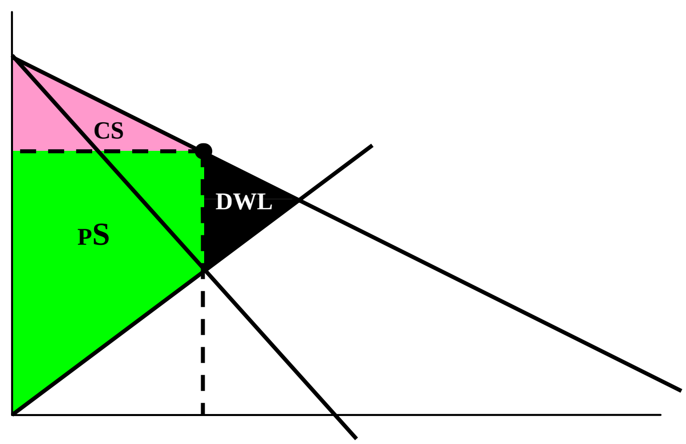
$$LI = \frac{p^M - MC^M}{p^M} 100\% = \frac{164.29 - 78.57}{164.29} 100\% = 52.18\%$$

Monopoly profit is

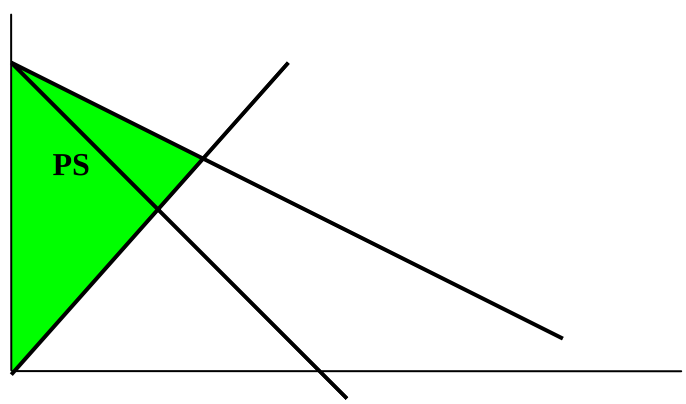
$$\pi = 250(28.57) - 3(28.57)^2 - 1250 - 50(28.57) - 0.5(28.57)^2 = \$1607.14$$

Price discrimination:

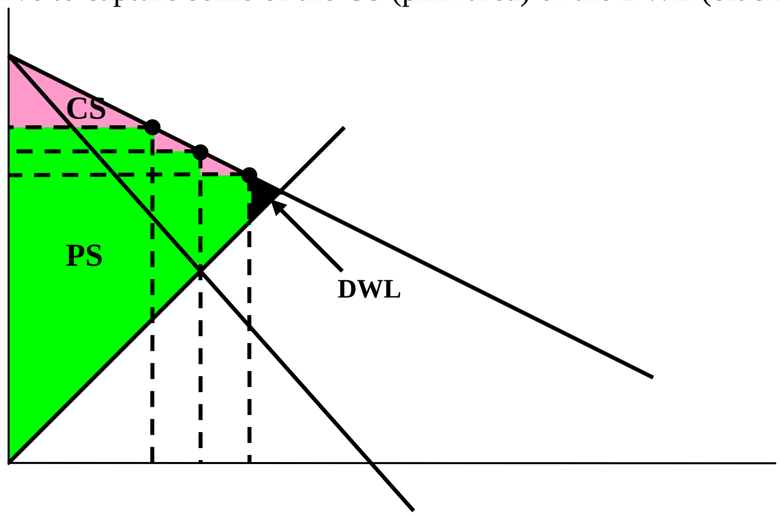
1. **Uniform monopoly pricing** involves charging all consumers the same price (see above)



2. In **first degree price discrimination** the monopolist charges each customer different prices to capture all of the CS and all of the DWL (no pink and black areas remain)

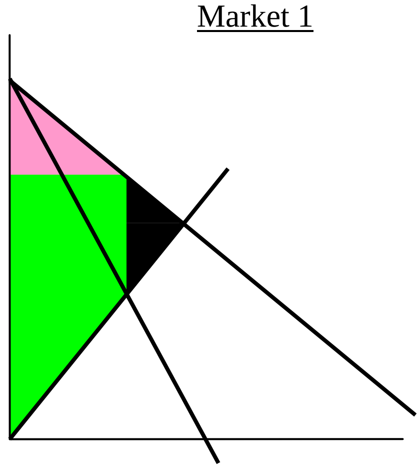


3. In **second degree price discrimination** the monopolist realizes it cannot charge each customer different prices, so it tries to charge two or more prices given the same demand curve to capture some of the CS (pink area) or the DWL (black triangle)

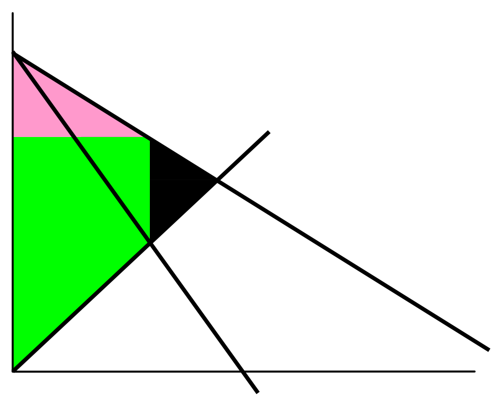


4. In **third degree price discrimination** the monopolist has been able to disaggregate the market demand curve into two or more separate demand curves, and it charges the uniform monopoly price in both of the disaggregated markets

Market 1



Market 2



Competitive Duopoly Example: Suppose that firms *A* and *B* form a duopoly in some market where firm *A* is not concerned with what firm *B* is doing and vice versa. Let

$$q_A \quad \text{and} \quad q_B$$

denote firm A and B's output, respectively. Suppose they face following **inverse demand curve**:

$$p = 250 - 3Q$$

and the *C* curves are

$$C_A = 625 + 50q_A + q_A^2 \quad \text{and} \quad C_B = 625 + 50q_B + q_B^2$$

Firms A and B solve the following problems in a contestable (competitive) duopoly:

$$\max \pi(q_A) = p \cdot q_A - \left(625 + 50q_A + q_A^2\right)$$

$$\max \pi(q_B) = p \cdot q_B - \left(625 + 50q_B + q_B^2\right)$$

Firm A's FOC is

$$\pi_q = \underbrace{p}_{MR} - \underbrace{\left(50 + 2q_A\right)}_{MC} = 0$$

$$p = 50 + 2q_A$$

$$q_A^* = \frac{p - 50}{2}$$

Firm B's FOC simplifies to

$$q_B^* = \frac{p - 50}{2}$$

Since firm B and firm A make up the entire market

$$Q^* = q_A^* + q_B^* = \frac{p - 50}{2} + \frac{p - 50}{2} = p - 50$$

$$Q^* = p - 50 \qquad (1)$$

Substituting this into the demand curve yields

$$p^* = 250 - 3\left(Q^*\right) = 250 - 3\left(p^* - 50\right)$$

$$p^* = 250 - 3p^* + 150$$

$$4p^* = 400$$

$$p^* = 100$$

Substitute this back into equation (1) or into the demand equation yields

$$Q^* = 100 - 50 = 50$$

or

$$100 = 250 - 3Q^*$$

$$3Q^* = 150$$

$$Q^* = 50 .$$

Another way to think about this problem is to convert the marginal cost curves (when *MC* = *p* these are the individual inverse supply curves) of the firm into the **inverse market supply curve**:

$$p = 50 + 2q_A$$

$$q_B = q_A = q$$

$$Q^S = 2q$$

$$q = 0.5Q^S$$

$$p = 50 + 2(0.5Q^S)$$

$$p = 50 + Q^S$$

The **supply curve** and **demand curve** are then

$$Q^S = p - 50$$

$$Q^D = \tfrac{250}{3} - \tfrac{1}{3}p$$

At the competitive equilibrium we have

$$Q^S = Q^D$$

$$p - 50 = \tfrac{250}{3} - \tfrac{1}{3}p$$

$$3p - 150 = 250 - p$$

$$4p = 400$$

$$p^* = 100 \quad \text{and} \quad Q^* = 50$$

The Lerner Index for this duopoly is

$$LI = \frac{P^* - MC}{P^*} 100\% = 0\%$$

The competitors in this duopoly produce

$$q_A^* = 25 \quad \text{and} \quad q_B^* = 25$$

Firm A and B profit are

$$\pi(q_A^*) = (100)\cdot(25) - 625 - 50(25) - (25)^2 = 0$$

$$\pi(q_B^*) = (100)\cdot(25) - 625 - 50(25) - (25)^2 = 0$$

Zero profits in a competitive duopoly discourage other firms from entering this market.

Cartel Example: two firms and zero profits encourage the firms to collude (form a cartel like OPEC). Because $q_A + q_B$ is equal to Q, the Cartel's total cost equation is

$$C = 625 + 50q_A + q_A^2 + 625 + 50q_B + q_B^2$$

Dropping the subscripts (firms are identical so $q_A = q_B = q$) yields

$$C = 2\cdot 625 + 50\cdot(q+q) + (q)^2 + (q)^2$$

$$C = 2\cdot 625 + 50\cdot(q+q) + 2\cdot \tfrac{2}{2}(q)^2$$

$$C = 2\cdot 625 + 50\cdot(2q) + \tfrac{1}{2}(2q)^2$$

$$C = 2\cdot 625 + 50\cdot(Q) + \tfrac{1}{2}(Q)^2$$

$$C = 2\cdot 625 + 50\cdot Q + 0.5Q^2$$

The Cartel's revenue equation is

$$R = (p)\cdot Q = (250 - 3Q)\cdot Q = 250Q - 3Q^2$$

The Cartel chooses Q to maximize its profit:

$$\max \pi(Q) = 250Q - 3Q^2 - \left[2\cdot 625 + 50\cdot Q + 0.5Q^2\right]$$

The FOC is $$\pi_Q = \underbrace{250 - 6Q}_{MR_{cartel}} - \underbrace{\left[50 + Q\right]}_{MC_{cartel}} = 0$$

Solving for Q yields

$$Q^M = \tfrac{200}{7} = 28.57$$

$$MC^M = 50 + 28.57 = \$78.57$$

$$MR^M = 250 - 6 \cdot 28.57 = \$78.57$$

$$P^M = 250 - 3 \cdot 28.57 = 164.29$$

The Lerner Index for this duopoly is

$$LI = \frac{P^M - MC^M}{P^M} 100\% = \frac{164.29 - 78.57}{164.29} 100\% = 52.18\%$$

Thus each member of the cartel agrees to produce

$$q_A^M = \tfrac{28.57}{2} = 14.285$$
$$q_B^M = 14.285$$

Firm A and B profit are

$$\pi(q_A^*) = (164.29) \cdot (14.285) - 625 - 50(14.285) - (14.285)^2 = \$803.57$$

$$\pi(q_B^*) = \$803.57$$

Is there an incentive to cheat on the cartel's output quota agreement? Suppose firm B does not cheat on its agreement of $q_B^M = 14.285$. If firm A knows this, then can it increase its profit by increasing output by 1 additional unit? First, how is the price affected by A's cheating?

$$P^M = 250 - 3 \cdot (28.57 + 1) = 161.29$$

Thus, firm A and B profits are

$$\pi(q_A^*) = (161.29) \cdot (14.285 + 1) - 625 - 50(14.285 + 1) - (14.285 + 1)^2 = \$842.44$$
$$\pi(q_B^*) = (161.29) \cdot (14.285) - 625 - 50(14.285) - (14.285)^2 = \$760.72$$

Firm B's profit is lower than it had expected while Firm A's is higher. Thus Firm B knows that Firm A cheated on its cartel production quota. Thus an incentive to cheat on the quota exits.

Cournot Market Example: Cournot Market is characterized by the following:

- Barriers to entry and exit

- Implicit collusion: firms make the same pricing decisions even though they have not consulted with one another (not illegal and common)
- Oligopolists' decisions are based on the decisions of their fellow oligopolies

Each firm supplies q_A and q_B, and so demand in the market is

$$p = 250 - 3(Q)$$

$$p = 250 - 3(q_A + q_B)$$

Given firm B's decision $\overline{q}_B$, firm A's revenue would be

$$R_A = q_A \cdot p$$

$$R_A = q_A \cdot (250 - 3q_A - 3\overline{q}_B)$$

$$R_A = 250q_A - 3q_A^2 - 3\overline{q}_B q_A$$

Firm A's chooses q_A given firm B's decision $\overline{q}_B$

$$\max \pi(q_A) = 250q_A - 3q_A^2 - 3\overline{q}_B q_A - \left[625 + 50q_A + q_A^2\right]$$

Firm A's FOC is

$$\pi_q = \underbrace{250 - 6q_A - 3\overline{q}_B}_{MR_A} - \underbrace{\left[50 + 2q_A\right]}_{MC_A} = 0$$

Solving this for q_A yields

$$q_A = \tfrac{200}{8} - \tfrac{3}{8}\overline{q}_B$$

Dropping the bar above q_B yields A's Best Response curve:

$$q_A^{Abr} = 25 - 0.375q_B$$

By symmetry, B's Best Response curve is

$$q_B = 25 - 0.375q_A$$

Solving this for q_A allows us to graph both curves together with q_A on the vertical axis:

$$q_A^{Bbr} = 66.7 - 2.667q_B$$

The **Cournot Equilibrium:**

$$q_A^{Abr} = q_A^{Bbr}$$

$$25 - 0.375q_B = 66.7 - 2.667q_B$$

$$2.291667q_B = 41.667$$

$$q_B^{CE} = \frac{41.6667}{2.2916667} = 18.1818$$

$$q_A^{CE} = 25 - 0.375\left(18.1818\right) = 18.1818$$

$$q_A^{CE} = 66.7 - 2.667\left(18.1818\right) = 18.1818$$

$$Q^{CE} = (18.1818)(2) = 36.3636$$

$$P^{CE} = 250 - 3(36.3636) = \$140.91$$

Thus firm profits are equal:

$$\pi(Q^{CE}/2) = (140.91)(18.18) - 625 - 50(18.18) - (18.18)^2 = \$697.22$$

Even though the firms in a Cournot market do not explicitly collude, both firms charge the same prices that are greater then the marginal cost of producing the 18.18th unit of output:

$$MC_A(q^{CE}) = 50 + 2(18.18) = 86.36$$

Thus the Lerner index is not equal to zero:

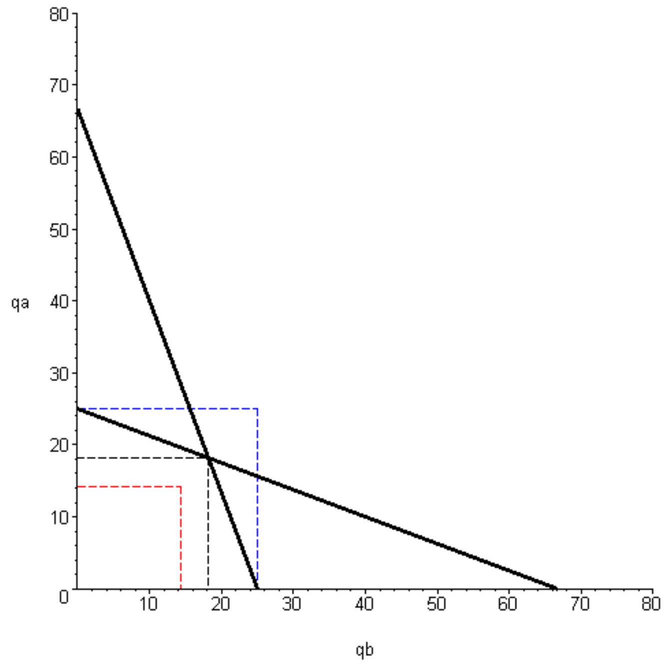
$$LI = \frac{140.91 - 86.36}{140.91}100\% = 38.7\%.$$

Thus, using this information, the government would incorrectly conclude that the firms in this duopoly are **explicitly** colluding to “jack” up prices (engage in price gouging).

Comparison of the Contestable, Cartel and Cournot equilibriums:

- The **cartel** produces the least amount of output at the highest price
- The **contestable (competitive) duopoly** produces the most output at the lowest price.
- Firm profit in each type of market is

duopoly	P	Q	q_A	q_B	π_A	π_B
Contestable	100	50	25	25	0	0
Cournot	140.91	36.36	18.18	18.18	697.22	697.22
Cartel	164.29	28.57	14.285	14.285	803.57	803.57



3. **Bertrand Equilibrium** is similar to the Cournot except the firm chooses its price given its competitor's chosen price.

4. **Stackelberg Equilibrium** is an equilibrium that arises when one firm announces its quantity (or price) first. Once this is done the other firms make their choices on quantity (or price)—LEADER-FOLLOWER.

Construct the Payoff Matrix for the Duopoly Game: Each firm chooses to produce 25, 18.18 or 14.285 units of output. There are 9 combinations of these output sets. Therefore there are 9 different market prices for each of these combinations and 9 different sets of firm profits. The prices and the profits for decision sets (25, 25), (18.18, 18.18), and (14.285, 14.285) are already computed. Thus we must compute the prices and profits for decision sets (18.18, 14.285), (18.18, 25), (14.285, 25):

$$p = 250 - 3(25 + 18.18) = 120.46$$
$$\pi(25) = (120.46)(25) - 625 - 50(25) - (25)^2 = \$511.50$$
$$\pi(18.18) = (120.46)(18.18) - 625 - 50(18.18) - (18.18)^2 = \$325.45$$

$$p = 250 - 3(25 + 14.285) = 132.15$$
$$\pi(25) = (132.15)(25) - 625 - 50(25) - (25)^2 = \$803.63$$
$$\pi(14.285) = (132.15)(14.285) - 625 - 50(14.285) - (14.285)^2 = \$344.38$$

$$p = 250 - 3(18.18 + 14.285) = 152.61$$
$$\pi(18.18) = (152.61)(18.18) - 625 - 50(18.18) - (18.18)^2 = \$909.85$$
$$\pi(14.285) = (152.61)(14.285) - 625 - 50(14.285) - (14.285)^2 = \$636.65$$

Firm A's output decision

Firm B's output decision	14.285	18.18	25
14.285	803.57 803.57	909.85 636.65	803.63 344.38
18.18	636.65 909.85	697.22 697.22	511.50 325.45
25	344.38 803.63	325.45 511.50	0 0

If Firm B decides to produce 14.285 units, then Firm A should produce 18.18 units—underline Firm A's profit of \$909.85 given these decisions. If Firm B decides to produce 18.18 units, then Firm A should produce 18.18 units—underline Firm A's profit of \$697.22 given these decisions. If Firm B decides to produce 25 units, then Firm A should produce 14.285 units—underline Firm A's profit of \$344.38 given these decisions.

If Firm A decides to produce 14.285 units, then Firm B should produce 18.18 units—underline Firm B's profit of \$909.85 given these decisions. If Firm A decides to produce 18.18 units, then Firm B should produce 18.18 units—underline Firm B's profit of \$697.22 given these decisions. If Firm A decides to produce 25 units, then Firm B should produce 14.285 units—underline Firm B's profit of \$344.38 given these decisions.

The above illustrates that neither firm has a dominate strategy. The cartel agreement (both firms producing 14.285 units of output) yields the maximum level of industry profit. However, Firm A has an incentive to cheat on this agreement. If it believes Firm B will not cheat, Firm A can generate \$909.85 in profit by producing 18.18 units of output. Similarly, Firm B can generate \$909.85 in profit by producing 18.18 units of output if it believes Firm A will not cheat. Thus both Firms cheat and both produce 18.18 units of output (the Cournot equilibrium), which yields lower firm and industry profits had neither firm cheated on its cartel quota—this why John Nash won the Noble Prize in economics!!!

If both profits are underlined in a box, then the corresponding decision set is a Nash Equilibrium (NE). Thus the decision set (18.18, 18.18) is a NE. Using the definition of NE yields this same conclusion:

q_A = 18.18 is optimal given q_B = 18.18
q_B = 18.18 is optimal given q_A = 18.18